

TAEUK KIM

ECE 521 HW 7

5

(C) is on the ~~back~~ last page

1. (a)

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 6 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$G(s) = C \cdot (sI - A)^{-1} \cdot B + D$$

$$(sI - A)^{-1} = \frac{1}{s^2 + s + 1} \begin{pmatrix} s+1 & 1 \\ -1 & s \end{pmatrix}$$

$$G(s) = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} s+1 & 1 \\ -1 & s \end{pmatrix} \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

$$= \frac{s+1}{s^2 + s + 1}$$

(b) By using MATLAB

$$L = \text{place}(A^T, C^T, p)^T = \begin{pmatrix} -6 \\ 11 \end{pmatrix}$$

$$\text{observer } oA = A - LC = \begin{pmatrix} 6 & 7 \\ 12 & -12 \end{pmatrix}$$

$$\text{|| } oB = B = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

$$\text{|| } oC = \cancel{C} = \begin{pmatrix} -6 \\ 11 \end{pmatrix}$$

$$2. G(s) = \frac{s^2 + 2s + 150}{s(s+4)(s+6)}$$

we have $f(s) = s^2 + 2s + 150$

$$A = \begin{pmatrix} -2 & -24 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad C = (1 \ 2 \ 50) \quad D = 0$$

set K for controller

$$K = \text{place}(A, B, sp)$$

system poles

$$K = (3, -9, 23)$$

set L for observer

$$L = \text{place}(A', C', op)$$

observer poles

$$L = \begin{pmatrix} -4.3077 \\ 8.4615 \\ 0.3077 \end{pmatrix}$$

3. as by using tf2ss,

$$[A, B, C, D] = \text{tf2ss}(b, a)$$

which are $b = [1]$;

$$a = [1 \ 1 \ 0];$$

system pole $sp = [-2+2i \ -2-2i]$

observer pole $op = [-8 \ -8]$

$$K = \text{place}(A, B, sp)$$

$$L = \text{place}(A^T, C^T, op)^T$$

observer based controller's A

$$\Rightarrow nA = A - BK - LC$$

make state space for us ss

$$(A) \text{ observer_state} = \text{ss}(nA, L, K, 0)$$

$$G(s) = \text{tf}(\text{observer_state})$$

$$= \frac{267s + 512}{s^2 + 19s + 117}$$

So,

$$\dot{\hat{x}} = nA\hat{x} + Bu$$

$$\dot{\hat{x}} = (A - LC - BK)\hat{x} + Ly$$

$$u = -K\hat{x}$$

$$\text{then } \Rightarrow \begin{cases} \dot{\hat{x}} = \begin{pmatrix} -4 & -57 \\ 1 & -15 \end{pmatrix} \hat{x} + \begin{pmatrix} 29 \\ 13 \end{pmatrix} y \\ u = -(3 \ 8) \hat{x} \end{cases}$$