ECE-528 HW #7

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1.

$E[X] = \int_0^1 3x^3\,dx$

$= \left[\frac{3}{4}x^4\right]_0^1 = \frac{3}{4}$

$E[Y] = \int_0^1 2y^2\,dy$

$= \left[\frac{2}{3}y^3\right]_0^1 = \frac{2}{3}$

$E[X^2] = \int_0^1 3x^4\,dx = \left[\frac{3}{5}x^5\right]_0^1$

$= \frac{3}{5}$

independent

$E[Y^2] = \int_0^1 2y^3\,dy$

$= \left[\frac{1}{2}y^4\right]_0^1 = \frac{1}{2}$

$Var[X] = \frac{3}{5} - \frac{9}{16} = \frac{3}{80}$

$var[Y] = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$

(a)

$E[Z] = E[X] - E[Y]$

$= \frac{3}{4} - \frac{2}{3} = \frac{9-8}{12} = \frac{1}{12}$

(b) $var(Z) = Var(X) + var(Y)$

$= \frac{3}{80} + \frac{1}{18} = \frac{67}{720}$

(c) $Z = X - Y$

$f_Z(z) = \int_0^1 f_X(x) f_Y(x-z)\,dx$

0 otherwise

$f_{X,Y}(x,y) = f_X(x) f_Y(y)$

$= \begin{cases} 6x^2y & 0 \le x < 1 \quad 0 \le y < 1 \\ 0 & \text{otherwise} \end{cases}$

$Z = X - Y \quad Y = X - Z \quad X = Z + Y$

$\max(Z) = 1 \quad \min(Z) = -1$

$-1 \le Z < 0$

$P(X - Y \le Z)$

$= \int_{-Z}^1 \int_0^{Y+Z} 6x^2y\,dx\,dy$

$0 \le Z < 1$

$P(X - Y \le Z)$

$= \int_0^1 \int_0^{Z} 6x^2y\,dx\,dy$

2. 4 possible time intervals

$\Rightarrow$ 0, 1, 2, 3, 4 $\Rightarrow Y$

Duration of task $\lambda(y)=1/(5-Y) \Rightarrow X$

$$E[Y] = \frac{0+1+2+3+4}{5} = 2$$

$$E[X|Y] = \frac{1}{\lambda(Y)} = 5-Y$$

$$E[X] = E[E[X|Y]] = E[5-Y]$$

$$= 5 - E[Y] = 3$$

(b) expected time at which the task is completed

$\Rightarrow$ duration of task & all time intervals

$E[X]+E[Y] = 3+2 = 5$

(c) possible time intervals

0, 1, 2, 3, 4, 5, 6 $\Rightarrow A$

professor's spending time while he met student. $\Rightarrow B$

C: student find professor

$\Rightarrow E[B|A]P(C)$

3 $E[T] = \mu \quad var(T) = \sigma_t^2$

$E[N_1] = 0 \quad var(N_1) = \sigma_1^2$

$E[N_2] = 0 \quad var(N_2) \quad \sigma_2^2$

$Y_1 = T + N_1$

$Y_2 = T + N_2$

zero mean $\Rightarrow E\{(T - aY)Y^T\} = 0$

$\Rightarrow E\{(T - (a_1Y_1 + a_2Y_2))Y_1\} = 0$

$E\{(T - (a_1Y_1 + a_2Y_2))Y_2\} = 0$

$\Rightarrow E\{-a_1Y_1^2 - a_2Y_1Y_2 + Y_1T\} = 0$

$E\{-a_2Y_2^2 - a_1Y_1Y_2 + Y_2T\} = 0$

$E[Y_1T] = E[a_1Y_1^2] + E[a_2Y_1Y_2]$

$E[Y_2T] = E[a_2Y_2^2] + E[a_1Y_1Y_2]$

$E[Y_1] = E[T] = \mu = E[Y_2]$

$E[Y_1^2] = E[Y_2^2] = [\mu^2 + \sigma_t^2 + \sigma_n^2]$

$\mu^2 = a_1\mu^2 + a_2(\mu^2 + \sigma_t^2 + \sigma_n^2)$

$\mu^2 = a_2\mu^2 + a_1(\mu^2 + \sigma_t^2 + \sigma_n^2)$

$a_1 = a_2 = \dfrac{\mu^2}{2\mu^2 + \sigma_t^2 + \sigma_n^2}$