$p \quad -1+1$

Taeuk Kim HW9

1.a) $E[X_n] = 1 \times P(H) + (-1) \times P(T)$ (head, Tail)

$P(H) = P(T) = \frac{1}{2}$

$E[X_n] = 0$

$R_X(n_1, n_2) = E[X_{n_1} X_{n_2}] - E[X_{n_1}]E[X_{n_2}]$

$= E[X_{n_1} X_{n_2}]$

$= (1)^2 \times P(H) + (-1)^2 \times P(T)$

$= 1$

mean is Constant and auto covariance is independent on time.

so, WSS

b) $Var(X_n) = E[X_n^2] - (E[X_n])^2$

$= 1$

$R_X(n_1, n_1) = 1$

$P(X_{n_1} = k_1, X_{n_2} = k_2, X_{n_3} = k_3)$

$$= \begin{cases} \frac{1}{2} & k_1 = k_2 = k_3 = 1 \\ -\frac{1}{2} & k_1 = k_2 = k_3 = -1 \\ 0 & \text{otherwise} \end{cases}$$

It is not dependent on sample difference

=> stationary

(c) Coin is biased

=> if $P(H) = p$, then $P(T) = 1 - p$

~~$P(X_{n_1} = k_1, X_{n_2} = k_2, X_{n_3} = k_3)$~~

doesn't mean moving time

It has cycle $X_n = X_{n+2}$, $X_{n+1} = X_{n+3}$

so it ~~also~~ cyclostationary

2. a) Head $X_n = -1\ 1\ -1\ 1 \cdots$

Tail $X_n = 1\ -1\ 1\ -1 \cdots$

$E[X_n] = (-1)^n \times \frac{1}{2} + (-1)^{n+1} \times \frac{1}{2} = 0$

$$R_X(n_1, n_2) = E[X_{n_1} X_{n_2}] = \begin{cases} -1 & \text{if } n_2 - n_1 = \text{odd} \\ 1 & \text{if } n_2 - n_1 = \text{even} \end{cases}$$

mean is constant, and autocovariance is independent on time

so, WSS

(b) $var(X_n) = 1 \times \frac{1}{2} + 1 \times \frac{1}{2} = 1$

if $P(X_{n_1} = k_1, X_{n_2} = k_2, X_{n_3} = k_3)$

$= P(X_{n_1+m} = k_1, X_{n_2+m} = k_2, X_{n_3+m} = k_3)$

then it is stationary.

stationary

$c = \frac{1}{2\pi}$ $\mu = \pi$

$var = \frac{4\pi^2}{12} = \frac{\pi^2}{3}$

c) X_n : $-1\ 1\ -1\ 1\ \cdots\ (-1)^n$

X_n : $1\ -1\ 1\ -1\ \cdots\ (-1)^{n+1}$

$\Rightarrow$ cycle $\Rightarrow$ cyclostationary

3. (a), (b)

If Y_n is stationary, it satisfys

$P[Y_{n_1} = y_1, Y_{n_2} = y_2, Y_{n_3} = y_3]$

$= P[Y_{n_1+r} = y_1, Y_{n_2+r} = y_2, Y_{n_3+r} = y_3]$

$P[Y_{n_1} = y_1, Y_{n_2} = y_2, Y_{n_3} = n_3]$

$= P[\frac{1}{2}(X_n + X_{n_1-1}) = y_1, \frac{1}{2}(X_n + X_{n_2-1}) = y_2, \frac{1}{2}(X_n + X_{n_3-1}) = y_3]$

$= P[\frac{1}{2}(X_1 + X_2) = y_1, \frac{1}{2}(X_{n_2-n_1+2} + X_{n_2-n_1+1}) = y_2, \frac{1}{2}(X_{n_3-n_1+2} + X_{n_3-n_1+1}) = y_3]$

4.

[I] = True $E[X(t)^2] = R_X(0)$

[II] = False

[III] $\Rightarrow$ We can't notice it is Gaussian distributed White Noise process can be exist if $X(t)$ isn't stationary Gaussian random process

$\Rightarrow$ False

[IV] $E[X(t)^2] = R_X(0)$

$\Rightarrow R_X(\tau) = 0$ for any $\tau \neq 0$

for any t_1, all samples are uncorrelated

$\Rightarrow$ True

5.

(a) $E[X(t)]$

$= E[A_1 \cos(w_0 t + \theta_1) + A_2 \cos(w_0 t + \theta_2)]$

$= E[A_1] \cos(w_0 t + E[\theta_1]) + E[A_2] \cos(w_0 t + E[\theta_2])$

uniform variable's mean and variance

$\mu = \frac{2\pi - 0}{2} = \pi$ $Var[\theta_1] = \frac{\pi^2}{3}$

$= E[A_1] E[A_2] (\cos(w_0 t + E[\theta_1]) + \cos(w_0 t + E[\theta_2])$

$\because E[\theta_1] = E[\theta_2]$

$= (E[A_1] + E[A_2]) (\cos(w_0 t + \pi))$

$R_{A_1A_2}(t_1, t_2) = R_{A_1A_2}(t_1 - t_2)$

(b) from the mean, mean is dependent on time t.

so $\boxed{\text{not stationary}}$ ← WS

(c) $A_1 \cos(w_0 t + \theta_1) + A_2 \cos(w_0 t + \theta_2)$

$\Rightarrow A_1 \cos(w_0(t+h) + \theta_1) + A_2 \cos(w_0(t+h) + \theta_2)$

cosine value is changed in period so,

$A_1 \cos(w_0 t + \theta_1) + A_2 \cos(w_0 t + \theta_2)$

$\neq A_1 \cos(w_0(t+h) + \theta_1) + A_2 \cos(w_0(t+h) + \theta_2)$

So, $\boxed{\text{not strictly stationary}}$